Macaroni, spaghetti, and physics

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This is an extended version of my talk at the "Primi d'Italia" National Festival held on 30 September 2023 in Foligno. This work represents a part of my long-standing popular science activity within the frameworks of "Culinary Universe".

"If Narcissus was turned into a flower,
I want to be metamorphosed into Macaroni."
Fellippo Sgruttendio, Neapolitan poet of the eighteenth century

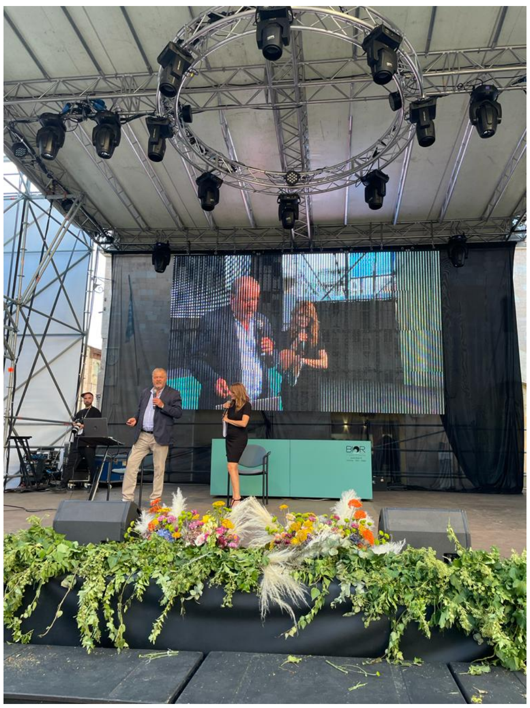

FIG. 1. Festival "Primi d'Italia" September 2023.

Each and everyone knows a lot about spaghetti, and many of you have cooked it at home. But have you ever thought about the physical processes taking place in the pot where pasta is cooking, and producing a correctly boiled "pasta" (in accordance to Italian standards) as a result? Have you ever asked yourself what happens inside spaghetti when it is floating in the boiling water? Why should we always follow the cooking time instructions specified on spaghetti packaging? Why is the cooking time different for different types of pasta? How does cooking time depend on the shape of pasta (classical spaghetti with different diameters, rigatoni or bucatini, etc.)? Does the cooking time depend on the location: are you making pasta at the seaside or in the highlands? When bent, why does an uncooked spaghetti strand ("spaghetto") almost never break into two pieces but into three or four? Why do spaghetti strands never tie into knots during cooking? How do you select the type of pasta, if you already have a sauce, so that the dish will be hot and tasty?

Below we will try to gain insight into the physics of the spaghetti cooking process and find the answers to some of the questions that may arise while the interested audience waits for a pot of pasta to boil in anticipation of a hearty dinner.

I. A GLIMPSE INTO A HISTORY OF PASTA AND ITS MANUFACTURING

Contrary to popular opinion, pasta was not brought to the West by Marco Polo after his trip to China (1295). In fact, its history began much earlier on the Mediterranean coast, at the time when the prehistoric man was leaving the nomadic lifestyle behind and started settling down and growing grain for food. The first flat cakes baked on top of hot stones were mentioned in the Old Testament (*Genesis* and *Kings*). In the first millennium B.C., Greeks were already making a thin layered pasta; they named it *"laganon"*. This word came to ancient Romans in the form of *"laganum"*, and may well have became the source for modern term *"lasagna"*.

Etruscans were also preserving thin layers of pasta. With the growth of the Roman Empire, pasta started spreading throughout Western Europe. Reliance on pasta as a way to preserve grain products emerged with the need to transfer food supplies during the periods of tribal migration. In Sicily, pasta was introduced by Arabs when they conquered the island in the tenth century. Sicilian pasta named *"trie"* may be considered the ancestor of spaghetti. It was shaped in thin strands, and the name originated from the Arabic word *"itryah"* (flat cake cut into strands). People living in Palermo started making pasta in the beginning of the second millennium. Based on a detailed will, probated by a Genoa notary public Ugolino Scarpa, we can positively argue that by 1280 macaroni products were already consumed in Liguria. It is known from the History of Italian Literature that pasta has attracted the attention of such writers as Jacopone da Todi, Cecco Angiolieri, and Felippo Sgruttendio. Ultimately, in Boccaccio's *Decameron* macaroni became a symbol of sophisticated gourmet food.

The first guilds of pasta makers (*"Pastai"*) with their own charters were created in the sixteenth century in Italy, where they received political and public recognition. At that time macaroni was regarded as a food for the rich, especially in the provinces which did not grow their own durum wheat (e.g., Naples). The invention of the mechanical press resulted in lowered production costs and hence the price of the product. As a result, by the seventeenth century pasta turned into one of the staple foods consumed by all social classes; now it is widespread in all countries of the Mediterranean basin. Naples has become a major center of pasta manufacturing and export. There, pasta with basil tomato sauce or sprinkled with grated cheese is sold on every street corner. In Northern Italy pasta became popular at the end of the eighteenth century mainly due to Pietro Barilla, who opened a small factory in Parma and later became a major producer in the Italian food industry.

Modern methods of pasta production are primarily based on the process of extrusion (pushing out through holes) and dragging. Extrusion was invented and used for the first time in the manufacture of long metal components with specified cross sections (see Figure 2). Extrusion processes are based on the property of fluidity and on subsequent pushing of material through a rigid die by means of compression. It can work both in cold and hot conditions. Dragging is a process similar to extrusion, the only difference being that in the case of dragging the material is pressed through the die located at the vessel's outlet and thus it becomes a process of stretching rather than compression. This method is used by the metal-processing industry for cylinder, wire, and pipe manufacturing. It allows reduction of the diameter of metal wire to 0.025 *mm*. Other materials that can be processed by extrusion are: polymeric compounds, ceramics, and food. Dies used to produce spaghetti are shown in Figure 3.

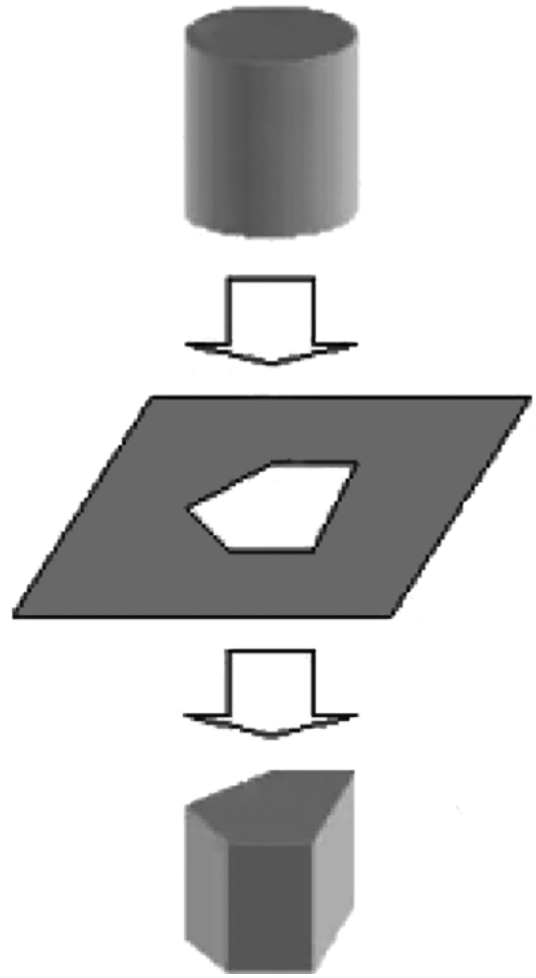

FIG. 2. Extrusion process is based on the property of fluidity and on subsequent pushing of material through a rigid die by means of compression.

FIG. 3. Spaghetti die.

II. A SCIENTIFIC WAY OF SPAGHETTI COOKING

Before we begin, let us derive the principal formula of culinary: the expression relating the cooking time of the boiled piece of meat with its size.

Suppose that a spherically symmetric homogeneous piece of meat (of the radius R) with an initial temperature T_0 is placed in an environment with a fixed temperature T_{ext} maintained. How much time does it take for the temperature of the meat in the center of the ball to reach the same temperature as the environment?

In mathematical physics, the process of heat transfer inside a sphere is described by a differential equation

$$\frac{\partial T(r,t)}{\partial t} = \left(\frac{\kappa}{\rho c}\right) \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T(r,t)}{\partial r} \right), \qquad (1)$$

where $T(r,t)$ is the temperature at a point r at time t, κ is the thermal conductivity of the meat, ρ is its density, and c is the specific heat. Since the water is boiling in a saucepan, the temperature at the surface of the sphere at any instant of time remains constant and equal to $T_{ext} = 100°C$:

$$T(r = R, t \geq 0) = 100°C. \quad (2)$$

We took the meat from the refrigerator, so at the time when it was dropped into the water, the temperature was $T_0 = 4°C$ throughout its volume:

$$T(0 \leq r \leq R, t = 0) = 4°C. \quad (3)$$

Eqs. (1)-(3) determine so-called problem of solution of differential equation with the boundary conditions. How to deal with them is well known for mathematicians and knowing the numerical values of the thermal conductivity of meat, its density and specific heat they will be able to accurately write a recipe for cooking broth.

Nevertheless, let us try to figure it out by ourselves using the dimensional analysis method. The temperature of denaturation of meat coincides by an order of magnitude with the boiling point of water (differs from it by $20 - 25\%$). Therefore, we assume that the time of "delivery" of the necessary temperature to the center of the solid sphere depends only on its material parameters and size: the thermal conductivity of the meat, its density, specific heat and radius. Therefore, we seek the dependence of the required time on the size of the sphere in the form:

$$\tau = \kappa^{\alpha} \rho^{\beta} c^{\gamma} R^{\delta}. \quad (4)$$

By comparing dimensions, we write:

$$[\tau] = [\kappa]^{\alpha} [\rho]^{\beta} [c]^{\gamma} [R]^{\delta}. \quad (5)$$

The dimension of the thermal conductivity $[\kappa] = \frac{kg \cdot m}{s^3 \cdot K}$. Substituting it, side by side with the dimensions of density ($[\rho] = \frac{kg}{m^3}$), specific heat ($[c] = \frac{J}{kg \cdot K}$) and radius ($[R] = m$), into Eq. (5) and then comparing them in the right and left hand sides, one finds: $\alpha = -1$, $\beta = \gamma = 1$, $\delta = 2$. Thus, we conclude that

$$\tau = C_0 R^2 / \chi, \quad (6)$$

where C_0 is an unknown constant of the order of unity and $\chi = \frac{\kappa}{\rho c}$ is called the coefficient of temperature conductivity. Substituting the quantities $\kappa = 0,45 \, \frac{W}{m \cdot K}$, $\rho = 1,1 \cdot 10^3 \, \frac{kg}{m^3}$, $c = 2,8 \cdot 10^3 \, \frac{kJ}{kgK}$ we find that for the meat $\chi = \kappa/(\rho c) = 1,4 \cdot 10^{-7} \, \frac{m^2}{s}$.

Consequently, a half kilogram piece of meat should be cooked for about an hour and a half. The estimate is in some way exaggerated, since we do not distinguish in its process the temperature of denaturation from the boiling point, but the order of magnitude is correct.

However, we ignored the time required for carrying out of the denaturation process itself. In most of cases, this time is so short that it can be neglected with respect to the "delivery" time. Yet, the connoisseurs of Chinese cuisine are familiar with another process which can be used to cook meat much faster. It is called in Chinese "Shuan" ("rinse" or "instant-boil"). It consists of soaking the thin-cut sliced beef or mutton in the boiling soup. Surprisingly that in only 10 seconds the sliced beef changes its color from pink to white or gray, indicating the slice is ready to eat. The beef slice becomes ready even while remaining between the chopsticks. To describe mathematically such process it is enough to replace in Eq. (6) the radius of the sphere by half of the thickness of the thin-cut slice L and to add the time required for denaturation process:

$$\tau_{\text{slice}} = \tau_{\text{denat}} + C_0 \frac{(L/2)^2}{\chi}. \quad (7)$$

Let us "recycle" Eqs. (6)-(7) for the process of pasta cooking. In the flour, molecules of starch are grouped into granules with diameter of 10–30 microns, which, in turn, are surrounded with different proteins. In the process of pasta fabrication two of them, gliadin and glutenin, combine with water, unite, and form a continuous net, called *gluten*, which is strong and has low permeability for water molecules. This net covers the starch granules. Cooking time is directly related to the capacity of the starch molecules (surrounded by gluten in the process of pasta drying) to absorb water, which begins penetrating through the gluten network and diffuses to the inward of pasta as soon as it is placed in the pan of boiling water. At a temperature of about $T_{\text{g}} = 70°C$, starch molecules begin forming a gel-like compound, which hinders water absorption. Pasta is considered "al dente" (to the tooth) the moment when the gel-like starch absorbs the minimum amount of water necessary to make it sufficiently soft. Hence in order to cook pasta it is necessary to deliver hot water inside the initially dry spaghetto.

Thus, one of the necessary conditions for the preparation of pasta is the same as when cooking meat: the temperature inside it should be increased to a certain specific value (in the case of pasta it is $70°C$). The second condition, in comparison to the boiling of meat is new one: the water here should penetrate inside the originally dry pasta. Both of these conditions have to be fulfilled during the cooking process. Indeed, neither the heating of dry spaghetti in the stove, nor their prolonged holding in cold water will lead to the appearance on the table of a plate of mouth-watering pasta.

The Eq. (6) was obtained as a result of the dimensional analysis of the heat transfer equation. Although we analyzed it in the simplest case of a spherically symmetric specimen, the result remains valid for a cylindrically symmetric "spaghetto". Moreover, fortunately, the process of water diffusion from the outside to the center of the "spaghetto" is described by the equation of the same type (the diffusion equation) as that one for the heat transfer, but the latter being written for the concentration of water instead of the local value of temperature. As a result, the spaghetto's cooking time is related to its diameter by the relation of the same type as Eq. (7):

$$\tau_{sp} = ad^2 + b. \tag{8}$$

where d is the spaghetto's diameter. The coefficient a is determined by the physical properties of the pasta (its temperature conductivity and, what is new, the diffusion coefficient), while the coefficient b characterizes ... the nationality of the eater. In fact, if the first term in the Eq. (8) determines the delivery time of water and heat to the center of the spaghetto, then the second tells us how long these factors affect the central part. That is why for Italians, who prefer to eat the pasta in a degree of readiness "al dente" ("on tooth"), the process of the gel formation of starch occurs not in the whole volume: the latter remains relatively firm at the central part of spaghetto. As a result, as we will see below from the analysis of experimental data, the coefficient b turns to be negative for them. In other countries, spaghetti lovers believe that the pasta should be well-boiled and the time that they cook pasta can significantly exceed the one indicated on the packaging by the Italian manufacturer.

The starch gelification temperature, $T_{\rm g}$, is constant while the boiling temperature of water, $T_{\rm b}$, depends on the height with respect to the sea level, H. Consequently, the coefficient $a = a\,(H)$ depends on H and hence, cooking time $\tau_{\rm sp} = \tau_{\rm sp}\,(H)$. The recommended cooking time on the packaging corresponds to the sea level, where $T_{\rm b} = 100°C$. At high altitudes, where water boils at lower temperatures, cooking time should be extended. In the extreme case of Everest (its height is 8,848 m) $T_{\rm b} = 73°C$, which is very close to $T_{\rm g}$ and pasta can be hardly cooked well at all.

Let us now go to a supermarket and buy every kind of cylindrically shaped pasta: capellini, spaghettini, spaghetti, vermicelli, bucatini. Read and collect in Table 1 the recommended cooking times (the column "Experimental cooking time". Then take a slide gauge, measure the corresponding diameters and fill out the same table (the column "diameter, external/internal"):

Table 1

Type of pasta	Diameter, external/internal (mm)	Experimental cooking time (min)
capellini no.1	1.15/-	3
spaghettini no.3	1.45/-	5
spaghetti no.5	1.75/-	8
vermicelli no.7	1.90/-	11
vermicelli no.8	2.10/-	13
bucatini	2.70/1	8

In order to find the numerical values of the coefficients a and b it is enough to write the equation (8) using the data from two rows of our Table 1:

$$t_1 = ad_1^2 + b \tag{9}$$

$$t_2 = ad_2^2 + b \tag{10}$$

and to solve the system of these two simple equations. As the reference case, we choose the data for spaghettini no. 3 and vermicelli no. 8. This gives:

$$a = \frac{t_2 - t_1}{d_2^2 - d_1^2} = 3.4\,\text{min}\,/mm^2$$

$$b = \frac{d_2^2 t_1 - d_1^2 t_2}{d_2^2 - d_1^2} = -2.3\,\text{min}\,.$$

We bought the Italian pasta, and the recommended cooking times on the packaging are given to obtain the "pasta al dente". Consequently, one can see that the coefficient b for Italians is indeed negative.

Having the numerical values of the coefficients a and b we can check how our formula works for other types of cylinder-shaped pasta. The results of our calculations can be found in Table 2 and show very good agreement with the experimental data for all rows except the two extremes: capellini and bucatini.

Table 2

Type of pasta	Experimental cooking time (min)	Theoretical cooking time (min)
capellini no.1	3	2.2
spaghettini no.3	5	5.0
spaghetti no.5	8	8.1
vermicelli no.7	11	10.0
vermicelli no.8	13	13.0
bucatini	8	22.5 ?!

This is a typical situation in theoretical physics: predictions of a theory have a range of validity corresponding to the assumed simplifications made for this theory. For instance, let us take a look at the last row of the table: for bucatini the difference between the theoretical and experimental values is striking: 22.5 min versus 8 min. This contradiction reflects an important fact: the range of possible thicknesses for all varieties of a whole cylinder-shaped pasta (capellini, spaghetti, vermicelli) is very narrow: 1 mm only. Indeed, our calculation of the "al dente" cooking time for bucatini in the approximation of the uniform cylinder gave 22.5 min, which would completely boil the periphery of such a thick "spaghettone" into mush.

The way to make such a thick spaghetti edible was found empirically: a hole should be made in the spaghetti strand along its axis. In the process of cooking, water enters through this hole, and there is no longer a need to deliver it to the core from the outside. One can try to modify the formula (8) deducting the internal diameter from the external one, and immediately the theoretical result comes close to the reality:

$$t = a\left(d_{\text{ext}} - d_{\text{int}}\right)^2 + b \approx 7.5\,\text{min}\,.$$

Nevertheless, one should keep in mind that the hole in the pasta cannot be less than ~ 1 mm in diameter, otherwise, due to capillary pressure,

$$P_{\text{cap}} = \frac{4\sigma\left(T = 100^{\circ}C\right)}{d_{\text{int}}} \sim 200\,Pa,$$

water will not be able to enter into it. The value 200 Pa corresponds to the pressure a couple of centimeters of water above the cooking pasta.

Another deviation of theory from reality occurs for very thin pasta. The reason for this error is obvious: speaking about cooking pasta "al dente" we have chosen the parameter b as negative: $b = -2.3\,\text{min}$. Formally, this means there should exist a pasta thin enough to not even need cooking at all to be eaten "al dente". The corresponding critical diameter d_{cr} can be determined from the relation:

$$\tau_{\text{cr}} = a d_{\text{cr}}^2 + b = 0,$$

which gives

$$d_{\text{cr}} = \sqrt{|b|/a} \approx 0.82\,mm.$$

One can see that the real diameter of capellini (1.15 mm) is not far from this critical value, so the underestimated capellini cooking time in Table 1 is the result of this limitation of our model.

III. SPAGHETTI KNOTTING

Cooked spaghetti strands entangled with each other present a complex tangle in hot water, however, the authors of this book have never seen them knot themselves. The reasons why this does not happen can be learned from a new field in statistical mechanics: the statistics of polymers.

The probability of a long polymer chain self-knotting is determined by the expression [1]

$$w = 1 - \exp\left(-\frac{L}{\gamma\xi}\right),$$

where L is the full length of a polymer, ξ is the characteristic length at which the polymer can change its direction by $\pi/2$ and $\gamma \approx 300$ is a large factor, obtained as a result of numerical and theoretical modeling. Applying this formula to spaghetti, where $\xi \approx 3$ cm, one can find the length $L_{\min}$ when the probability of self-knotting becomes noticeable ($w \sim 0.1$) :

$$\exp\left(-\frac{L_{\min}}{\gamma\xi}\right) \approx 0.9$$

which gives

$$L_{\min} \approx \gamma\xi \ln 1.1 \approx 30\xi \approx 1\ m.$$

The length of a standard spaghetto is 23 cm and this is not long enough to form knots.

IV. THE SECRETS OF MIXING OF PASTA WITH SAUCE

The rules of good etiquette prescribe to start eating at the same time everyone is sitting at the table. However, at the Italian table, an exception is made for the first dish, pasta: it is eaten immediately, as the plate appears on the table. The pasta should be hot. However, it gets to the table not directly from a pot of boiling water: first it is discarded in a colander and mixed with the sauce. It is clear that this process takes some time and can be long. Then the paste will cool down and the pleasure will be destroyed.

In order to understand what time is required to mix the sauce and pasta, we start with a simple model: let the viscous liquid flow through the cylinder under the action of gravity (this is the simple model of the pasta in which the sauce flows). The stationary flow ($Q = \Delta M/\Delta t$) of a liquid in a pipe of diameter D under the effect of pressure difference ΔP is determined by the Poiseuille formula

$$Q = \frac{\pi\rho\Delta P}{2^7\eta l}D^4,$$

where η is the viscosity of the fluid, ρ is its density, and l is the pipe length. On the other hand, the magnitude of the flow is determined by the mass of fluid flowing through the cross section of the pipe per unit time:

$$Q = \Delta M/\Delta t = \frac{1}{4}\pi\rho D^2 \Delta l/\Delta t.$$

Comparing these two expressions and assigning the pressure difference to the effect of the gravitational force $\Delta P = \rho g \Delta l$ we find

$$\frac{\Delta t}{\Delta l} = \frac{32}{\rho g}\left(\frac{\eta}{D^2}\right).$$

We see that the rate at which a viscous liquid fills the tube is proportional to the ratio η/D^2.

This formula is obtained in the model of fluid flow in a gravitational field through a vertically arranged tube, so the acceleration of free fall g is included in the answer. Yet, it is clear that the nature of the acceleration is insignificant: with the same success, g in this formula can be replaced by the acceleration which pasta acquires from being stirred by a ladle.

Above we have considered liquid flowing inside the pipe. Nevertheless, it is clear that if several tubes are tightly arranged side by side, the liquid between the tubes will flow more or less at the same speed as inside them.

Thus, we arrive at the conclusion that the characteristic time of pasta stirring is

$$\tau_{\rm mix} \sim \left(\frac{\eta}{D^2}\right).$$

The greater the viscosity of the sauce, the larger the diameter of the pasta should be. It would be hard to mix well the finest capellini with a viscous "pesto genovese": the latter naturally combines with short "trofie". And *vice versa*, liquid tomato sauce just drains to the bottom of the plate being mixed with huge "paccheri". So it is better to leave the cherry tomatoes in it, and also dress the sauce with pieces of zucchini and shrimps.

FIG. 4. Different types of Italian pasta: trofie, paccheri.

The obtained formula also helps us to understand the empirical rules of Italian cuisine. Usually, pasta and sauce are prepared at the same time, the pasta is cooked in a saucepan, and the sauce in a frying pan. In the hands of a good chef, they arrive to the state of readiness almost at the same time. Discarding the pasta in a colander and decanting the water, he sends pasta to the pan, where it mixes with the sauce. Viscosity drops noticeably with increasing temperature, so the mixing time of the pasta with the sauce boiling in the frying pan will be significantly shorter than if the sauce was taken out of the refrigerator. In addition, the pasta does not lose heat during its mixing with the sauce. Another subtlety. The pasta should be boiled in a saucepan for slightly less time in comparison with the time recommended on the package. Being stirred with sauce in a pan it continues to be cooked and "gathers additionally" the missing minutes in order to achieve the "al dente" condition.

V. BREAKING A SPAGHETTI STRAND

In the beginning of this chapter, we mentioned another interesting property of spaghetti related to its mechanical fracture. Take both ends of a spaghetto and bend it into an arch, gradually increasing its curvature. One can suppose that sooner or later it will break into two parts somewhere near the middle. It turns out, though, that in this case our intuition is not correct: nearly always it will break into three or more pieces.

Such unusual behavior of a spaghetti strand attracted the attention of numerous scientists, Richard Feynman among them. But only a short time ago in 2005, owing to the research studies conducted by the two French physicists Audoly and Neukirch, was a quantitative description of this phenomenon obtained.

The scientists studied the behavior of a thin, elastic rod under the effect of flexural deformation. They wrote the differential equation which describes the distribution of the tension (so-called Kirchhoff equation) in a curved elastic rod, first with both ends fixed. Then they studied what will happen to the tension distribution along the rod after the instantaneous release of one of its ends. Only a numeric solution was obtained, but it provided an understanding of the essence of the process. Qualitatively, the explanation is as follows.

Let us suppose that due to the applied mechanical stress, the first fracture occurs at some (weakest) point of the strand. It would seem that after being broken, both parts of the rod should return to their equilibrium positions.

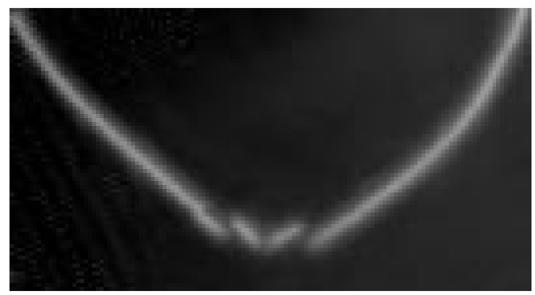

FIG. 5. Instant photo of spaghetti's fracture.

This is true, however the transition to the equilibrium state occurs in a very nontrivial way. The first fracture generates flexural waves in both fragments of the rod which start to propagate along each of the fragments. Evidently, flexural waves of this kind (generated by the first fracture) will dampen out with time, but, at certain ratios between the rod length and its elastic modulus, the wave propagation can lead to a subsequent rod fragmentation. Indeed, propagation of such a wave means periodic growth and decrease of the local flexural stress along the rod. It is important to note that these flexural waves propagate on the background of the already existing initial homogeneous bend of the rods, which relaxes much more slowly than the flexural wave period. As a result of the summation of these two quasistatic and dynamic stresses, further rod fractures may occur at other points, where such sums exceed a critical value. It is also noteworthy that after intricate computations, the researchers confirmed their theoretical conclusions by filming the experimental spaghetti break studies with a high-speed camcorder (Figure 5).

Very recently, Ronald Heisser and Vishal Patil, two students of the Massachusett Institute of Technology (USA), working on a scientific project proposed to them by their tutor Jörn Dunkel, managed to overcome the magic reluctance of the spaghetto to break into two parts. For this, as it turned out, it is enough to twist spaghetto, say 360 degrees, along its axis. In this case, after the first break, only part of its released energy is spent on the flexural wave excitation, the rest into spinning the spaghetto (excitation of the "twisting wave"). As a result, the amount of released energy is no longer sufficient for the second break...

While the spaghetti is cooking, you can entertain yourself with several remaining dry spaghetti by experimentally verifying the findings of Audoly and Neukirch.

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[1] The author is grateful to A.Y. Grosberg for introduction to the theory of knots.